

Final Exam

May 4, 2007

1 Proofs

- 1 Prove that in a simple graph with at least two vertices, there must be vertices with the same degree. (20 points)

2 Binary Problems

2 Fill in the missing blanks (1 point each). Assume 8 bits for the two's complement representation.

Decimal	Binary	Two's comp	Hexadecimal
-46	_____	_____	_____
_____	-110111	_____	_____
_____	_____	11111110	_____
_____	_____	_____	1F

3 Logic Statements

3 Provide the truth table for the statement $(q \vee r) \leftrightarrow p$ (3 points)

4 Show the equivalent statement without the outermost negation of $\neg(q \wedge \neg q)$ (4 points)

5 Show the equivalent statement without the outermost negation of $\neg(\exists p, \forall q, S(p, q))$. (4 points)

4 Sets

Use sets A and B in problems 6-8 Let $A = \{w, x, y, z\}$ and $B = \{v, w\}$.

6 Compute $A \cup B$ (2 points)

7 Compute the power set of B . (4 points)

8 Compute $A \times B$. (6 points)

5 Matrices

Use A and B in problems 9-11

$$\text{Let } A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \text{ Let } B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{bmatrix}$$

9 Compute $A+B$ (2 points)

10 Put B in row-echelon form (4 points)

11 Find the inverse of A. To get credit, you must show ALL steps. (6 points)

6 Probability

Providing the formula is sufficient.

12 Given a 3x3 matrix, and three symbols, a, b, and c, how many different arrangements of 3 a's, 2 b's and 4 c's are possible? (3 points)

13 What is the expected value when rolling 100 6-sided die? (2 points)

14 In one draw, what is the probability of drawing exactly one heart in 4 draws from a deck of 52 cards? Assume you replace the card after each draw. There are 13 hearts in a deck.(5 points)

15 How many different permutations of letters are there from the words "Discrete Structures"? You may ignore the space. (5 points)

For problems 16-18, assume $A = \{w, x, y, z\}$ and $R = \{(w, w), (w, x), (x, x), (x, y), (y, y), (y, z), (z, w), (z, x)\}$.

16 Find the symmetric closure of R.(3 points)

17 Find the transitive closure of R. (5 points)

18 Place an 'X' in the blank if R satisfies the property. (1 point each)

_____irreflexive _____asymmetric _____antisymmetric

7 Trees and Graphs

19 Draw the first 4 states that might be explored in a depth first search of a tic-tac-toe game tree.(5 points)

20 Draw the first 4 states that might be explored in a breadth first search of a tic-tac-toe game tree. (5 points)