

Exam IV

May 4, 2007

1 Vocabulary

One point for each blank.

- 1 _____ is an edge connecting a vertex with itself
- 2 if there is an edge between two vertices they are _____
- 3 if a vertex is an endpoint of an edge, the edge is _____ with the vertex
- 4 a vertex of degree zero is _____
- 5 a vertex of degree one is _____
- 6 _____ is a path of length $n \geq 1$ that begins and ends at the same vertex
- 7 a _____ path is a path that contains every edge of a graph exactly once
- 8 a _____ circuit is a circuit in a simple graph that passes through each vertex exactly once
- 9 _____ is an undirected graph with no simple circuits (not necessarily connected)
- 10 _____ is a vertex with no children
- 11 _____ is a listing of the vertices of a tree
- 12 _____ a tree containing all vertices of a graph
- 13 _____ is the form of an expression obtained from a preorder traversal of the tree representing the expression
- 14 _____ is an algorithm that optimizes by making the optimal choice at each step
- 15 _____ is a procedure for constructing a spanning tree that successively adds all edges incident to the last set of edges added, unless a simple circuit is formed

2 Proofs

16 Prove that every connected graph with n vertices has at least $n - 1$ edges.

3 Computations

Problems 17 nad 21 use the graph represented by this adjacency matrix.

*	A	B	C	D	E	F	G	H	I
A	0	2	7	0	0	0	0	0	0
B	2	0	0	2	0	0	0	0	0
C	7	0	0	2	0	7	0	0	0
D	0	2	2	0	0	0	0	0	0
E	0	0	0	0	0	3	3	0	0
F	0	0	7	0	3	0	0	2	1
G	0	0	0	0	3	0	0	5	0
H	0	0	0	0	0	2	5	0	0
I	0	0	0	0	0	1	0	0	0

- 17 Use Dijkstra's algorithm (you must demonstrate you are using this algorithm) to find the shortest path between node A and node G

18 Find the Euler path in the graph.

19 One edge can be added to create a Euler cycle. (a) What is that edge and (b) why is it the ONLY single edge that can be added to create the Euler cycle?

20 (Extra credit) Describe a conversation that Hamilton and Euler might have about this graph. Include some details about their lives.

21 Use Prim's algorithm (you must demonstrate that you are using this algorithm) to find the minimum spanning tree for the graph.