Definitions. (1 pt each)

1. ____________ is an ordered arrangement of the elements of a set

2. ____________ the study of arrangements of objects

3. ____________ when more than k objects are placed in k boxes, there must be a box containing more than one object

4. ____________ the counting of arrangements of objects

5. ____________ an unordered selection of r elements of a set

6. ____________ a subset of the sample space of an experiment

7. ____________ events E and F such that \( p(E \cap F) = p(E)p(F) \)

8. ____________ the weighted average of a random variable, with values of the random variable weighted by the probability of the outcomes

9. ____________ an experiment with two possible outcomes

10. ____________ the set of possible outcomes of an experiment

11. ____________ the representation of the composite of relations R and S

12. ____________ the representation for the inverse of a relation R

13. ____________ a set of elements called vertices and ordered pairs of these elements

14. ____________ a path that begins and ends at the same vertex

15. ____________ a relation that is reflexive, antisymmetric, and transitive
For problems 16-25, assume there are 30 students in the class. Provide the formula for the answer. You may also provide a numeric answer, but it is not required. 5 pts each.

16. 10 students are selected at random to answer a question. Assuming a student can be asked multiple questions, how many different ways can students be asked a question?

17. 10 students are selected at random to answer a question. After answering, the student leaves the room. Now how many different ways can students be asked questions?

18. 5 students are selected at random for a class demonstration. How many different groups of students are possible?

19. For the previous question, how many different groups are possible if the same student can be selected more than once?

20. A student was totally unprepared for a 20-question multiple choice exam. As a result, he had to randomly guess which of the four answers would be correct. What is the probability that the student will make an A (i.e., get 18 or more questions correct)?

21. For the previous question, how many questions is the student expected to get right?
22. How many different arrangements of numbers are possible from my phone number
   254-772-6671?

23. This is a three-step problem. We have two true random number generators that generate
   numbers between 0 and 99 (inclusive – meaning both 0 and 99 can be generated). Call
   these random number generators R and S. Both are equally likely to generate any number
   in its range. (10-15 points total)
   
a. What is the probability that R will generate a number less than 10?

b. What is the probability that the sum of R and S will be a number less than 10?
   Note: This portion of the problem is extra credit. You can answer if you want, or
   you can see me, and I will give you the answer. You get no credit if I provide the
   answer, but part c is difficult to get correct if this part is wrong.

c. What is the probability that the sum of R and S will be a number less than 10
   given that R has generated a number less than 10?

24. You are a contestant on the game show, “Deal or No Deal.” There are 5 cases remaining,
   and the cash values are $100, $200, $300, $400, and $10,000. What is the expected value
   for playing this game?
For problems 25-31, the set \( A = \{1,2,3,4\} \), and \( R \) is a relation on \( A \times A \).

25. Assume \( R = \{(1,1),(1,2),(1,3),(1,4)\} \), place an X in the blank if \( R \) satisfies the property. (3 pts)

- _____ reflexive _____ irreflexive _____ symmetric
- _____ asymmetric _____ antisymmetric _____ transitive

26. Assume \( R = \{(1,1),(1,2),(1,3),(1,4),(2,2),(2,1),(3,3),(3,1),(4,4),(4,1)\} \), place an X in the blank if \( R \) satisfies the property. (3 pts)

- _____ reflexive _____ irreflexive _____ symmetric
- _____ asymmetric _____ antisymmetric _____ transitive

27. Assume \( R = \{(1,2),(2,3),(3,1),(3,4),(4,1),(4,2),(4,3),(4,4)\} \). Provide the matrix representation of the relation. (4 pts)

28. For the same \( R \) as in problem 28, provide the directed graph representation of the relation. (4 pts)
29. For the same R as in problem 28, what is added by the reflexive closure? (4 pts)

30. For the same R as in problem 28, what is added by the symmetric closure? (4 pts)

31. Provide an example relation R (over the same A) that is both symmetric and antisymmetric. (4 pts)

32. For the same R as in problem 28, use Warshall’s algorithm to find the transitive closure. (9 pts)
33. Extra Credit: Although Blaise Pascal died when James Bernoulli was only 8 years old, they shared some common interests. Provide a hypothetical conversation the two could have had.