

# Exam I

CSI 2350

January 30, 2007

## 1 Word Lists

One point for each blank.

1. A \_\_\_\_\_ is a statement that is true or false but not both.
2. A \_\_\_\_\_ uses a logical and for a connective, while a \_\_\_\_\_ uses a logical or for a connective.
3. A conditional statement consists of a \_\_\_\_\_ that implies a \_\_\_\_\_.
4. An implication  $p \rightarrow q$  is logically equivalent to its contrapositive \_\_\_\_\_.
5. Two statements are logically equivalent if their \_\_\_\_\_ are the same.
6. \_\_\_\_\_ is the symbol for universal quantification.
7. A sequence of statements that end with a conclusion is an \_\_\_\_\_.
8. \_\_\_\_\_ are forms of incorrect reasoning.
9. The only difference between a \_\_\_\_\_ and a \_\_\_\_\_, is that the first are used to prove theorems.
10. \_\_\_\_\_ is the formal term for a statement that is generally believed to be true, but has not been proved.
11. In 2's complement, adding two negative numbers and getting a positive number is an example of \_\_\_\_\_.
12. Octal is a base \_\_\_\_\_ number system.

## 2 Binary Problems

13. Fill in the missing blanks (2 points each). Assume 8 bits for the two's complement representation.

Decimal	Binary	Two's comp	Hexadecimal
_____	1110111	_____	_____
_____	_____	11100111	_____
_____	_____	_____	-7B

### 3 Word Problems

Provide the propositional logical statements that are equivalent to the English sentences. Use  $p$  – it is raining,  $q$  – we went to the store,  $r$  – we bought apples from our neighbor. (3 points each)

14. If it is raining, then we won't go to the store.

15. It was raining, but we went to the store anyway.

16. If we didn't buy apples from our neighbor and we didn't go to the store, it was raining.

Solve the logic puzzle (5 points).

17. There are two types of people on the island, knights, who always tell the truth, and knaves who always lie. You meet three people standing in the road, and you know they are not all knights or all knaves. The first says "Exactly one of the others is a knave." The second says "Exactly one of us is a knight." Who are knights and knaves? Show your work!

18. Is the following argument valid (4 points)? Some Martians are green. Venusians eat some green things. Therefore, Venusians eat Martians. Defend your answer!

## 4 Logical Equivalences

Provide the truth table for the following statement. (4 points)

19.  $p \leftrightarrow (q \vee r)$

Provide the negation of each statement. (3 points each)

20.  $p \rightarrow q$

21.  $\forall p, \exists q, S(p, q)$ .

Provide the logically equivalent statement (3 points each)

22.  $p \vee (p \wedge q) \equiv$  \_\_\_\_\_ (Absorption)

23.  $p \wedge p \equiv$  \_\_\_\_\_ (Idempotent)

Provide the conclusion indicated. (3 points each).

24.  $(\neg q) \wedge (p \rightarrow q)$  therefore \_\_\_\_\_ (Modus tollens)

25.  $(p \vee q) \wedge (\neg p \vee r)$  therefore \_\_\_\_\_ (Resolution)

26.  $(p \rightarrow q) \wedge (q \rightarrow r)$  therefore \_\_\_\_\_ (Hypothetical Syllogism)

## 5 Proofs

6 points for each proof.

27. Provide a direct proof to show that  $\forall a \in \mathcal{I}, a^2 + (a + 1)^2$  is odd.

28. Provide an indirect proof to show that  $\forall a \in \mathcal{I}, a^2$  is odd  $\rightarrow a$  is odd.

29. Provide a proof using cases to show that if  $p > 0$  and  $p$  is a multiple of 3, and  $q > 0$  and  $q$  is a multiple of 5, then 15 is the smallest number that can be  $p$  and  $q$ .

30. Use contradiction to prove that if a circle has area  $A$  and circumference  $C$  then a circle with circumference  $C/2$  has area less than  $A/2$ .

31. Extra Credit! Write 100 **interesting** words about one of the people discussed in chapter 1.