

Exam II

March 1, 2007

1 Vocabulary

One point for each blank.

1. _____ is a graphical representation of a set or sets.
2. _____ or an onto function, is a function from A to B such that every element of B is the image of some element in A.
3. _____ is the set of elements in the universal set that are not in the set A.
4. _____ is a function with a domain that is a subset of the set of integers
5. _____ is the number of elements in a set
6. _____ is an algorithm that makes the best choice at each step.
7. _____ is the amount of time for an algorithm to solve a problem
8. _____ is a positive integer greater than 1 with exactly two positive integer divisors.
9. _____ is a type of encryption where the encryption keys are public knowledge, but the decryption keys are kept secret.
10. _____ is the matrix obtained from A by interchanging the rows and columns
11. _____ is the proof of $P(1)$ in a proof by mathematical induction of $\forall nP(n)$.
12. _____ is the principle that every non-empty set of integers has a least element.
13. _____ is an algorithm that proceeds by reducing a problem to the same problem with smaller input
14. _____ is a property that remains true during every traversal of a loop
15. _____ is the proof of $P(k) \rightarrow P(k + 1)$ for all positive integers k in a proof of $\forall nP(n)$.

20. Compute AB (5 points)

21. Put B in row-echelon form (5 points)

22. Find the inverse of A . (8 points)

4 Function Notation

2 points for each correct X. WARNING! Guessing can be very bad on these problems.

23. Place an 'X' by each function that is in $O(x^2)$.

_____ 2^x _____ $207.3x + 1045$ _____ $20x^2 + x/10$ _____ $x^3/1000$ _____ $\log_2(x)$

24. Place an 'X' by each function that is in $\Omega(x^2)$.

_____ 2^x _____ $207.3x + 1045$ _____ $20x^2 + x/10$ _____ $x^3/1000$ _____ $\log_2(x)$.

25. Place an 'X' by each function that is in $\theta(x^2)$.

_____ 2^x _____ $207.3x + 1045$ _____ $20x^2 + x/10$ _____ $x^3/1000$ _____ $\log_2(x)$

26. List 5 common big-O classes used in computer science. Put them in order ascending order. (5 points)

27. Use the Extended Euclidean Algorithm to find the inverse of 2 modulo 17. Show your work!

5 Proofs and Algorithms

10 points for each proof and algorithm.

28. Provide by induction that

$$\sum_i^n \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$$

29. Prove by strong induction that if you have a 3 cent stamp and a 10 cent stamp, you can represent all values of postage over 27 cents.

30. Write a recursive algorithm to find a^{2^n} , where a is a real number and n is a positive integer.

31. Extra Credit! Who were the inventors of the RSA algorithm? The more you can tell me about them, the more credit you will get.