

Name _____

1. Find the weakest precondition for the following conditions and statements.
(15 points)

<pre> if (x>y) { z = x; } else { z = x + y; } </pre>	<pre> if (a==b && c>b) { q = c; } else { q = a + b; } </pre>	<pre> if (a<b) { c = b - a; } else { c = a - b + 1; } </pre>
$\{z > y\}$	$\{q > a \wedge q > b\}$	$\{c > 0\}$

2. Find reasonable loop invariants for the following loops.
(10 points)

<pre>x = n; y = 0; while (x < 0) { x = x - 1; y = y + x + 1; }</pre>	<pre>i = 0; p = 0; t = 0; while (i < n) { p = A[i] * A[i]; t = t + p; i = i + 1; }</pre>
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3. Find post-conditions that will imply the correctness of the following three code segments.
(15 points)

<pre>// put max of L into L[0] for (k=1 ; k<Size; k++) { if (L[0]<L[k]) { T = L[0]; L[0] = L[k]; L[k] = T; } }</pre>	<pre>// Make A[i] larger than the // corr. element of B for (k=1 ; k<Size; k++) { if (A[k]<=B[k]) { A[k] = B[k] + 1; } }</pre>	<pre>//Find median of a, b, c. if (a>b) { t = a ; a = b; b = t; } if (b>c) { t = b ; b = c; c = t; } if (a>b) { t = a ; a = b; b = t; } m = b;</pre>
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4. Translate the following statements into universally or existentially quantified statements. Assume that the arrays B and C have indexes that run from 1 to 10.
(10 points)
- a. Every element of B is contained somewhere in C.
 - b. C contains two elements that add up to 25.
 - c. B doesn't have any negative elements.
 - d. The sum of any two corresponding elements of B and C is 75.
 - e. C has an element that is smaller than anything in B.

5. Given the following statements and post conditions, find the weakest precondition.
(10 points)

Statement	Condition	Weakest Precondition
$i = i + 1;$	$(\forall k : 1 \leq k \leq i : M \geq B[k])$	
$a = b + z;$	$z > b$	
$y = 3 * x * x;$	$(\exists y : 1 \leq y < n : B[y] > 12)$	
$B[0] = 12;$	$(\forall n : 1 \leq n \leq x : B[n] > C[n])$	
$a = b + 3$	$a \leq b \leq c \leq d$	

6. Prove or disprove the following proposition.
(10 points)

$$(a \wedge (\neg b \vee c)) = ((\neg a \vee b) \Rightarrow (a \wedge c))$$

7. For each pair of conditions, A and B, mark the weakest.
(10 points)

Weakest	Condition A	Weakest	Condition B
	$A \wedge B$		A
	$A \vee B$		A
	$2 < P < 10$		$0 < P < 75$
	$(\exists x : 1 \leq x < 10 : d[x] = x^2)$		$(\exists x : 1 \leq x < 9 : d[x] > x^2)$
	$(\forall x : 1 \leq x < 5 : c[x] > d[x])$		$(\forall x : 1 \leq x < 50 : c[x] > d[x])$
	$(\exists x : 1 \leq x < m : c[x] = b[x] + d[x])$		$(\forall x : 1 \leq x < m : c[x] = b[x] + d[x])$
	$B \wedge (A \Rightarrow B)$		$A \wedge (A \Rightarrow B)$
	$(A < B) \wedge (A < C)$		$A < B < C$
	$A < 100$		$A \leq 100$
	$75 < 100$		$A > 100$