Name_____

1. For each pair of conditions mark the weakest.
   (10 points)

Weakest	Condition A	Weakest	Condition B
	$(A < B) \wedge (C < D)$		$(A < B)$
	$Z > 0$		$Z > 10$
	$Q < 3$		$Q < 13$
	$(Ex : 1 \le x < 10 : b[x] > 30)$		$(Ex : 1 \le x < 20 : b[x] > 30)$
	$(Ax : 1 \le x < 10 : b[x] > 30)$		$(Ax : 1 \le x < 20 : b[x] > 30)$
	$(Ax : 1 \le x < 10 : b[x] > 30)$		$(Ex : 1 \le x < 10 : b[x] > 30)$
	A where $A \Rightarrow B$		B where $A \Rightarrow B$
	$(A < B) \vee (C < D)$		$(A < B)$
	$A < B$		$A \le B$
	$Z = Z + 1$		$A + B = B + A$

2. Translate the following statements into universally or existentially quantified statements. Assume that the arrays B and C have indexes that run from 1 to 10. (10 points)
   a. B has an element greater than 5.
   b. Every element of B is greater than zero.
   c. The first five elements of B are less than 10.
   d. The sum of every consecutive pair of elements of C is equal to 7.
   e. The array C contains the square of every element of B, but not necessarily in the correct order. In other words, for every i, $C[j] = (B[i])^2$ for some j, but i and j are not necessarily equal.

3. In the following expressions x is either bound or free. Write the word "bound" or "free" in each blank, depending on whether x is bound or free. (10 points)

_____a. $x + 1 = 2y + 3$

_____b. $(Ax : 1 \le x < y : x > y)$

_____c. $(Ay : 1 \le y < x : 2x + y = p + z)$

_____d. $(Ex : 1 \le x < 7 : b[i] = 0)$

_____e. $xy = 2x + 1$

4. Perform the following substitutions.

a. E^{y}_{x+y} where $E = (An : 1 \le n < k : b[n] = 2x + y)$

b. E^{x}_{y} where $E = (An : 1 \le n < k : b[n] = 2x + y)$

c. E^{z}_{z*z} where $E = (An : 1 \le n < k : b[n+z] = 2z + 1)$

d. E^{k}_{j+1} where $E = (An : 1 \le n < k : b[n] = 2k + n)$

e. E^{t}_{2*t+s} where $E = (At : 1 \le t < k : b[t] = t)$

5. Given the following pairs of statements and conditions, find the weakest precondition. (10 points)

Statement	**Condition**	**Weakest Precondition**
x:=x+1;	x>25	
x:=x+y+z;	z>x	
x:=2*x+1;	$(x > y) \wedge (x < z)$	
x:=z+2;	y>z	
x:=2*p;	$(An : 1 \le n \le k : b[n] = x + n)$	
x:=13+12*y;	$(An : 1 \le n \le x : b[n] > z)$	
x:=2*j+k;	$(x > y) \Rightarrow (k > j)$	
x:=y;	x>y	
x:=17;	x<25	
x:=2x+1	$(Ax : 1 \le x \le k : p[x] > 17)$	