

NAME _____

1. Which of the following are legal fully parenthesized propositions?

Write *yes* or *no* in each blank.

a. _____ $(b \wedge b \wedge b)$

f. _____ $(a \wedge b) \vee c) \wedge d)$

b. _____ $(c \wedge (c \wedge c))$

g. _____ $\neg(\neg(\neg(\neg(\neg((b \wedge c))))))$

c. _____ $(((((((($

h. _____ $p \Rightarrow q \Rightarrow r$

d. _____ $a = b$

i. _____ $((a \vee a) \vee (a \vee a))$

e. _____ $((a \neq b) \vee (a = b))$

j. _____ $(a \wedge b)$

2. Give the truth tables for the following propositions.

a	b	c	$(a \wedge b) \vee (a \wedge c)$	$(a \vee b) \wedge (a \vee c)$	$(a \vee b) \Rightarrow (a \vee c)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

3. Use the laws of Boolean algebra to prove the following equality. *Use only one law for each step!* There is no need to fill the entire diagram.

$$\neg(a \wedge \neg(b \wedge c)) = ((b \vee \neg a) \wedge \neg(a \wedge \neg c))$$

Step	Law Applied (from cheat-sheet)
$\neg(a \wedge \neg(b \wedge c))$	<i>Given</i>

4. Prove the following using the natural deduction system.
 $((c \Rightarrow d) \wedge (c \Rightarrow e)) \wedge c \Rightarrow (d \wedge e)$

5. Translate the following statements into universally or existentially quantified statements. Assume that the arrays B and C have indexes that run from 1 to 10.
- B has two different elements that are equal.
 - Every element of C is greater than 3.
 - The first five elements of B are less than the third element of C.
 - The first three elements of C are all less than the last seven elements of C.
 - The array C has an element that is larger than anything in B.

6. Perform the following substitutions.

a. E_{x+n}^y where $E = (An : 1 \leq n < k : b[n] = 2x + y)$

b. E_y^x where $E = (Ax : 1 \leq x < k : b[x] = 2y)$

c. E_{z*z}^z where $E = (En : 1 \leq n < k : b[n] = y^2)$

d. E_{j+1}^k where $E = (An : 1 \leq n < k : b[n] = 2m + n)$

e. E_{2p+q}^p where $E = (Ex : 1 \leq x < k : b[x] = 4p)$