

Name \_\_\_\_\_

1. Which of the following are legal fully parenthesized propositions?  
Write *yes* or *no* in each blank. (10 points)

a. \_\_\_\_\_  $(a \wedge (b \wedge c))$

f. \_\_\_\_\_  $((a \wedge b) \vee c) \wedge d$

b. \_\_\_\_\_  $q \Rightarrow p$

g. \_\_\_\_\_  $(\neg b \Rightarrow c)$

c. \_\_\_\_\_  $(a = b)$

h. \_\_\_\_\_  $((p, q) \Rightarrow r)$

d. \_\_\_\_\_  $(p \Rightarrow q) \Rightarrow p$

i. \_\_\_\_\_  $(a \wedge (\neg a))$

e. \_\_\_\_\_  $((a = b) \wedge (c = d))$

j. \_\_\_\_\_  $ab + cd$

2. Give the truth tables for the following propositions. (10 points)

$a$	$b$	$c$	$(c \wedge \neg a) \Rightarrow b$	$(a \wedge b) \vee (b \wedge c)$	$\neg a = (b \vee c)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

3. Use the laws of Boolean algebra to prove the following equality. *Use only one law for each step!* There is no need to fill the entire diagram. (10 points)

$$(\neg q \Rightarrow \neg p) = (p \Rightarrow q)$$

Step	Law Applied (from cheat-sheet)
$(\neg q \Rightarrow \neg p)$	<i>Given</i>

4. Prove the following propositions using truth tables. (10 points)
- $$(\neg c \vee (\neg a \wedge b)) = ((a \vee \neg b) \Rightarrow \neg c) \quad (a \wedge b) \Rightarrow (a \vee b)$$

5. Prove the following using the natural deduction system. (10 points)
- $$((A \wedge B) \vee (C \wedge D)) \Rightarrow (A \vee D)$$