

Are all integers equal?

Suppose that x and y are two integers such that $x \neq y$. Now consider the following argument.

Suppose that

1. $A = B$

Then

2. $xA = xB$

and

3. $yA = yB$

Of course we also have:

4. $yB = yA$

Adding equations 2 and 4 we get:

5. $xA + yB = xB + yA$

Now let's move the x 's to the left and the y 's to the right. This gives:

6. $xA - xB = yA - yB$

Now let's factor out the x 's and the y 's.

7. $x(A - B) = y(A - B)$

Now let's cancel out $(A - B)$.

8. $x = y$

But we assumed that $x \neq y$!

OK, what went wrong?