

1. Graphs

a. Terminology

Define: Vertex, Node, Edge, Arc, Path, Adjacent, Simple Path, Cycle, Simple Cycle,

b. Relations

Show the graph of the following relationship: (a,b), (b,c), (b,d), (b,e), (c,f), (c,g), (e,h).

c. Euler Paths

Draw a graph that has an Euler cycle.

Draw a graph that has an Euler path.

Draw a graph that has neither.

d. Bipartite

What is a bipartite graph?

2. Counting

a. Multiplication principles.

I have three urns with numbered balls in them. The first has three red balls, the second has four green balls, and the third has two white balls. How many ways are there to choose one white ball, one red ball and one green ball?

b. Unions and Intersections

3. Permutations and Combinations

a. Binomial theorem

in $(x+1)^{20}$, what is the coefficient of x^{18} ?

b. Algorithms

What is the next permutation after 54123?

Five things taken three at a time, what is the next combination after 135?

4. Probability

a. Equally likely events

Suppose I roll two dice and flip two coins. What are the equally likely outcomes of this experiment?

b. Adding disjoint sets

If we choose a number n from 1 through 100, what is the probability that $n \equiv 0 \pmod{5}$? What is the probability that $n \equiv 2$ or $3 \pmod{5}$?

c. Adding non-disjoint sets

If we choose a number n from 1 through 100, what is the probability that n is divisible by 5? What is the probability that n is divisible by 2? What is the probability that n is divisible by 5 or by 2?

5. Prove that:

$$\sum_{i=1}^n (4i + 7) = 2n^2 + 9n$$

6. Prove that:

$$\sum_{i=0}^n 5^i = \frac{5^{n+1} - 1}{4}$$

7. Multiply the following:

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 2 \end{pmatrix}$$

8. Multiply the following:

$$\begin{pmatrix} 0 & 2 & 3 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

9. Combine the following:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 1 & 2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 6 & 5 & 4 \end{pmatrix}$$

10. How fast are the following algorithms?

BubbleSort, Quicksort, Mergesort, Repeated Minimum sort, Insertion sort.

11. Convert from binary into hexadecimal.

11101011101001 1001011101110 101010011 1000000100001

12. Give addition and multiplication tables for Modulo 5 arithmetic.

13. Which of the following are propositions.

- Eat at Joe's!
- Make me smile.
- I'm in love with a big blue frog.
- If cows were green then you couldn't see them in the grass.
- Did you eat yet?
- Money is the root of all evil.

14. Which of the following are true?

- If cows are green then pigs can fly.
- If $x > 7$ then $x + 5 > 12$
- If 3 is a prime number then 10 is odd.
- If I won the lottery then I will go to Paris.
- If 9 is a prime number then 10 is bigger than 100.

15. Given $U=\{1,2,3,4,5,6,7,8,9,10\}$, $A=\{1,2,3\}$, $B=\{4,5,6\}$, $C=\{6,7,8\}$ give the following.

- a. $A \cap B$
- b. $A \cup B$
- c. $B \cap C$
- d. \overline{A}
- e. $B - C$

16. State and prove De Morgan's laws for sets.

17. Give the commutative, associative, and distributive laws for Boolean Algebra.

18. Give the identity and complement laws for Boolean Algebra.

19. What is an equivalence relation?

20. What is a partial order?

21. Negate

- a. $(\forall x)x > 7$
- b. $(\forall x)x < x + 1$
- c. $(\exists x)x < 3 \wedge x > 7$

22. Prove that $(x+1)^3 = x^3 + 3x^2 + 3x + 1$

23. Find the minimal Boolean Equations for the following truth tables.

ABCD	F	ABCD	F
0000	0	1000	0
0001	0	1001	0
0010	0	1010	0
0011	0	1011	1
0100	1	1100	0
0101	1	1101	1
0110	0	1110	0
0111	1	1111	1

ABC	G
000	1
001	1
010	1
011	0
100	1
101	0
110	0
111	0

24. Convert the following expression to prefix notation.

$$((a+b)*c)/(e/f)+(((c-d)-e)*(a+b))$$