

1. Prove that:

$$\sum_{i=1}^n (4i + 7) = 2n^2 + 9n$$

2. Prove that:

$$\sum_{i=0}^n 5^i = \frac{5^{n+1} - 1}{4}$$

3. Multiply the following:

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 2 \end{pmatrix}$$

4. Multiply the following:

$$\begin{pmatrix} 0 & 2 & 3 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

5. Combine the following:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 1 & 2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 6 & 5 & 4 \end{pmatrix}$$

6. How fast are the following algorithms?

BubbleSort, Quicksort, Mergesort, Repeated Minimum sort, Insertion sort.

7. Convert from binary into hexadecimal.

11101011101001 1001011101110 101010011 1000000100001

8. Give addition and multiplication tables for Modulo 5 arithmetic.

9. Which of the following are propositions.

- Eat at Joe's!
- Make me smile.
- I'm in love with a big blue frog.
- If cows were green then you couldn't see them in the grass.
- Did you eat yet?
- Money is the root of all evil.

10. Which of the following are true?
- If cows are green then pigs can fly.
  - If  $x > 7$  then  $x + 5 > 12$
  - If 3 is a prime number then 10 is odd.
  - If I won the lottery then I will go to Paris.
  - If 9 is a prime number then 10 is bigger than 100.
11. Given  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{1, 2, 3\}$ ,  $B = \{4, 5, 6\}$ ,  $C = \{6, 7, 8\}$  give the following.
- $A \cap B$
  - $A \cup B$
  - $\overline{B \cap C}$
  - $\overline{A}$
  - $B - C$
12. State and prove De Morgan's laws for sets.
13. Give the commutative, associative, and distributive laws for Boolean Algebra.
14. Give the identity and complement laws for Boolean Algebra.
15. What is an equivalence relation?
16. What is a partial order?
17. Negate
- $(\forall x)x > 7$
  - $(\forall x)x < x + 1$
  - $(\exists x)x < 3 \wedge x > 7$
18. Prove that  $(x+1)^3 = x^3 + 3x^2 + 3x + 1$

19. Find the minimal Boolean Equations for the following truth tables.

| ABCD | F | ABCD | F |
|------|---|------|---|
| 0000 | 0 | 1000 | 0 |
| 0001 | 0 | 1001 | 0 |
| 0010 | 0 | 1010 | 0 |
| 0011 | 0 | 1011 | 1 |
| 0100 | 1 | 1100 | 0 |
| 0101 | 1 | 1101 | 1 |
| 0110 | 0 | 1110 | 0 |
| 0111 | 1 | 1111 | 1 |

| ABC | G |
|-----|---|
| 000 | 1 |
| 001 | 1 |
| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 0 |
| 110 | 0 |
| 111 | 0 |