

NAME: \_\_\_\_\_

1. Prove that:

$$\sum_{i=1}^n i2^i = (n-1)2^{n+1} + 2$$

2. Combine the following permutations. Remember  $f \circ g(x) = f(g(x))$ .

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 2 & 3 & 5 \end{pmatrix}$$

3. Convert the following numbers from decimal to binary.
- 25
  - 37
  - 1024
  - 14

4. Give a minimal Boolean expression for the following function, F.

TTT	T
TTF	F
TFT	F
TFF	F
FTT	T
FTF	T
FFT	F
FFF	T

5. I toss three coins. What are the probabilities of the following events?
- I get three heads.
  - I get at least one head.
  - I get at least one head and at least one tail.
  - I get exactly two tails.

6. Give the graph of the following relation:  $(a,b), (b,a), (b,c), (c,d), (e,f), (f,a), (c,a)$ .

7. I have an urn with three red balls, four white balls and seven green balls. I draw a ball, look at it, write down its color, and drop the ball back into the urn. What is the probability of the following events:
- I draw three green balls in a row?
  - I draw a green ball followed by a red ball?
  - I draw a ball that is not green?
8. In the polynomial  $(x+1)^{12}$ , what is the coefficient of  $x^3$ ?

9. In the list of all permutations of the digits 1,2,3,4,5 what follows 4,5,1,2,3? In the list of all combinations of five digits from the set  $\{1,2,3,4,5,6,7,8\}$ , what follows 12678?

10. Decrypt the following message, and have a Merry Christmas. (You do not have to finish this here. You may remove this page and take it home. (Automatic 10 points for this question.)

GUFOFH TOF GUF PTOJ URYBFPH BUKPP ARLF DTN K  
BRAV; WFUTPJ, K LROARV BUKPP XTVXFRLF, KVJ  
WFKO K BTV, KVJ BUKPP XKPP URB VKYF RYYKVNFP  
-- ATJ IRGU NB.

RBKRKU 7:14