

NAME: \_\_\_\_\_

(All questions worth 10 points.)

1. Prove that:

$$\sum_{i=1}^n 2^i + 4i = 2(2^n + n^2 + n) - 1$$

2. Find a Boolean Expression for the following function  $f$ .

A	B	C	D	$f$
T	T	T	T	F
T	T	T	F	F
T	T	F	T	F
T	T	F	F	F
T	F	T	T	F
T	F	T	F	T
T	F	F	T	T
T	F	F	F	T
F	T	T	T	F
F	T	T	F	T
F	T	F	T	T
F	T	F	F	T
F	F	T	T	F
F	F	T	F	T
F	F	F	T	T
F	F	F	F	T

3. Multiply the following matrices.

$$\begin{pmatrix} 3 & 5 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 6 & 1 \\ 1 & 3 \end{pmatrix}$$

4. Multiply the following matrices.

$$\begin{pmatrix} 1 & 2 & 2 \\ 3 & 7 & 1 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 8 & 1 & 5 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

5. Combine the following permutations as indicated. Remember that  $(f \circ g)(x) = f(g(x))$ .

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 1 & 3 & 4 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 1 & 2 \end{pmatrix}$$

6. Combine the following permutations as indicated.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 6 & 1 & 7 & 2 & 5 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 1 & 3 & 4 & 2 & 5 \end{pmatrix}$$

7. Give the addition and multiplication tables for Modulo 9.

Addition:

+	0	1	2	3	4	5	6	7	8
0									
1									
2									
3									
4									
5									
6									
7									
8									

7. Continued  
Multiplication:

$\times$	0	1	2	3	4	5	6	7	8
0									
1									
2									
3									
4									
5									
6									
7									
8									

8. Prove the following algebraically:  $(x+3)^3 = x^3 + 9x^2 + 27x + 27$ . Apply one law at a time, and at each step *apply only one law*.

9. Prove the following:

$$A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$$

10. Prove that the following is true for all integers  $a$  and  $b$ .

$$-a \bullet -b = ab$$